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Algebra 1

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Problem Set #5

Exercise 1 :
If $S \neq \varnothing$ subset of G then $W_S = \{a_1 \dots a_r : r < \infty, a_r \in S \cup S^{-1}\}$ is a subgroup and is equal $< S >$.

Exercise 2 :
In $(\mathbb{Z}/12\mathbb{Z}, +)$, determine the subgroup H generated by :

1. $[2]$.
2. $[3]$.
3. $[2]$ and $[3]$.

Exercise 3 :
Prove that if H is a subgroup of $(\mathbb{Z}, +)$, $\exists m \geqslant 0$ in $\mathbb{Z}$ such that $H = m\mathbb{Z}$.

Exercise 4 :
In $(\mathbb{Z}/12\mathbb{Z}, +)$, find all $[k]$ that are cyclic generators with respect to $(+)$. We are looking for $a = [k]$ with additive order $o(a) = |\mathbb{Z}/12\mathbb{Z}| = 12$.

Exercise 5 :
Suppose a group element $x \in (G, \cdot)$ has the property $x^m = e$ for some integer $m \neq 0$. Then x has finite order $o(x)$, but the exponent m might not be the order $o(x)$ of the element x. Prove that any such exponent m must be a multiple of $o(x)$. (Hint : Letting $s = o(x)$, write $m = qs + r$ with $0 \leqslant r < s$).

Exercise 6 :
Prove that $(U_8, \cdot)$ is not cyclic. Prove that $(U_7, \cdot)$ is cyclic.

Exercise 7 :
If a group G is generated by a subset S, prove that any homomorphism $\phi : G \to G'$ *is determined by what it does to the generators*, in the following sense :

> *If $\phi_1, \phi_2 : G \to G'$ are homomorphisms such that $\phi_1(s) = \phi_2(s)$ for all $s \in S$, then $\phi_1 = \phi_2$ everywhere on G.*

This can be quite useful in constructing homomorphisms of G, especially when the group has a single generator.